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Conformally flat supergeometry in five dimensions

Sergei M. Kuzenko and Gabriele Tartaglino-Mazzucchelli

School of Physics M013, The University of Western Australia,
35 Stirling Highway, Crawley W.A. 6009, Australia
E-mail: kuzenko@cyllene.uwa.edu.au, gtm@cyllene.uwa.edu.au

Abstract: Using the superspace formulation for the 5D $\mathcal{N} = 1$ Weyl supermultiplet developed in arXiv:0802.3953, we elaborate the concept of conformally flat superspace in five dimensions. For a large family of supersymmetric theories (including sigma-models and Yang-Mills theories) in the conformally flat superspace, we describe an explicit procedure to formulate their dynamics in terms of rigid 4D $\mathcal{N} = 1$ superfields. The case of 5D $\mathcal{N} = 1$ anti-de Sitter superspace is discussed as an example.

Keywords: Extended Supersymmetry, Superspaces, Supergravity Models.

In the context of the two-brane Randall-Sundrum scenario [1] and its supersymmetric extensions [2−4], it is of interest to have a superspace description for five-dimensional $\mathcal{N}=1$ conformally flat supergeometry that would be similar to that available in the case of four-dimensional $\mathcal{N}=1$ supersymmetry, see, e.g. [5] for a review. This is also an interesting problem from the point of view of formal supergravity. Such a description can be derived using the superspace formulation for the Weyl multiplet of 5D $\mathcal{N}=1$ conformal supergravity [6, 7], which has recently been given in [8] (building on [9, 10]). Its elaboration is provided in the present letter. The case of 5D $\mathcal{N}=1$ anti-de Sitter superspace, which was studied in [11] from a different perspective, is explicitly worked out as an example.

To start with, it is worth recalling the salient points of the superspace formulation developed in [8]. Let $z^{\hat{M}} = (x^{\hat{m}}, \theta^{\hat{\mu}}_i)$ be local bosonic (x) and fermionic (θ) coordinates parametrizing a curved five-dimensional $\mathcal{N}=1$ superspace $\mathcal{M}^{5|8}$, where $\hat{m}=0,1,\dots,4$, $\hat{\mu}=1,\dots,4$, and $i=\underline{1},\underline{2}$. Here the Grassmann variables $\theta^{\hat{\mu}}_i$ obey the standard pseudo-Majorana reality condition $\overline{\theta^{\hat{\mu}}_i} = \theta^i_{\hat{\mu}} = \varepsilon_{\hat{\mu}\hat{\nu}}\,\varepsilon^{ij}\,\theta^{\hat{\nu}}_j$ (see the appendix in [10] for our 5D notation and conventions). The tangent-space group is chosen to be $\mathrm{SO}(4,1)\times\mathrm{SU}(2)$, and the superspace covariant derivatives $\mathcal{D}_{\hat{A}} = (\mathcal{D}_{\hat{a}}, \mathcal{D}^i_{\hat{\alpha}})$ have the form

$$\mathcal{D}_{\hat{A}} = \mathcal{E}_{\hat{A}} + \Omega_{\hat{A}} + \Phi_{\hat{A}}\ . \tag{1}$$

Here $\mathcal{E}_{\hat{A}} = \mathcal{E}_{\hat{A}}{}^{\hat{M}}(z)\,\partial_{\hat{M}}$ is the supervielbein, with $\partial_{\hat{M}} = \partial/\partial z^{\hat{M}}$,

$$\Omega_{\hat{A}} = \frac{1}{2}\,\Omega_{\hat{A}}{}^{\hat{b}\hat{c}}\,M_{\hat{b}\hat{c}} = \Omega_{\hat{A}}{}^{\hat{\beta}\hat{\gamma}}\,M_{\hat{\beta}\hat{\gamma}}\,, \qquad M_{\hat{a}\hat{b}} = -M_{\hat{b}\hat{a}}\,, \quad M_{\hat{\alpha}\hat{\beta}} = M_{\hat{\beta}\hat{\alpha}} \tag{2}$$

is the Lorentz connection,

$$\Phi_{\hat{A}} = \Phi_{\hat{A}}{}^{kl}\,J_{kl}\,, \qquad J_{kl} = J_{lk} \tag{3}$$

is the SU(2)-connection. The Lorentz generators with vector indices ($M_{\hat{a}\hat{b}}$) and spinor indices ($M_{\hat{\alpha}\hat{\beta}}$) are related to each other by the rule: $M_{\hat{a}\hat{b}} = (\Sigma_{\hat{a}\hat{b}})^{\hat{\alpha}\hat{\beta}} M_{\hat{\alpha}\hat{\beta}}$ (for more details, see the appendix of [10]). The generators of $\mathrm{SO}(4,1)\times\mathrm{SU}(2)$ act on the covariant derivatives as follows:[1]

$$[J^{kl}, \mathcal{D}^i_{\hat{\alpha}}] = \varepsilon^{i(k}\mathcal{D}^{l)}_{\hat{\alpha}}\,, \quad [M_{\hat{\alpha}\hat{\beta}}, \mathcal{D}^k_{\hat{\gamma}}] = \varepsilon_{\hat{\gamma}(\hat{\alpha}}\mathcal{D}^k_{\hat{\beta})}\,, \quad [M_{\hat{a}\hat{b}}, \mathcal{D}_{\hat{c}}] = 2\eta_{\hat{c}[\hat{a}}\mathcal{D}_{\hat{b}]}\,, \tag{4}$$

where $J^{kl} = \varepsilon^{ki}\varepsilon^{lj}J_{ij}$.

The covariant derivatives obey (anti)commutation relations of the general form

$$[\mathcal{D}_{\hat{A}}, \mathcal{D}_{\hat{B}}\} = \mathcal{T}_{\hat{A}\hat{B}}{}^{\hat{C}}\mathcal{D}_{\hat{C}} + \frac{1}{2}\mathcal{R}_{\hat{A}\hat{B}}{}^{\hat{c}\hat{d}}M_{\hat{c}\hat{d}} + \mathcal{R}_{\hat{A}\hat{B}}{}^{kl}J_{kl}\,, \tag{5}$$

where $\mathcal{T}_{\hat{A}\hat{B}}{}^{\hat{C}}$ is the torsion, $\mathcal{R}_{\hat{A}\hat{B}}{}^{\hat{c}\hat{d}}$ and $\mathcal{R}_{\hat{A}\hat{B}}{}^{kl}$ the SO(4,1) and SU(2) curvature tensors, respectively.

To describe the Weyl multiplet of conformal supergravity [6, 7], the torsion has to obey the constraints [8]:

$$\mathcal{T}^{ij}_{\hat{\alpha}}{}^{\hat{c}}_{\hat{\beta}} \;=\; -\,2\mathrm{i}\varepsilon^{ij}(\Gamma^{\hat{c}})_{\hat{\alpha}\hat{\beta}}, \qquad \mathcal{T}^{ij}_{\hat{\alpha}}{}^{\hat{\gamma}}_{\hat{\beta}k} \;=\; \mathcal{T}^{i}_{\hat{\alpha}\hat{b}}{}^{\hat{c}} \;=\; 0, \qquad \mathcal{T}_{\hat{a}\hat{b}}{}^{\hat{c}} \;=\; \mathcal{T}_{\hat{a}\hat{\beta}(j}{}^{\hat{\beta}}{}_{k)} \;=\; 0\ . \tag{6}$$

[1]The operation of (anti)symmetrization of n indices is defined to involve a factor $(n!)^{-1}$.

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With the constraints introduced, it can be shown that the torsion and the curvature tensors in (5) are expressed in terms of a small number of dimension-1 tensor superfields, $\mathcal{S}^{ij}$, $\mathcal{X}_{\hat{a}\hat{b}}$, $\mathcal{N}_{\hat{a}\hat{b}}$ and $\mathcal{C}_{\hat{a}}{}^{ij}$, and their covariant derivatives, with the symmetry properties:

$$\mathcal{S}^{ij}=\mathcal{S}^{ji}\,,\qquad \mathcal{X}_{\hat{a}\hat{b}}=-\mathcal{X}_{\hat{b}\hat{a}}\,,\qquad \mathcal{N}_{\hat{a}\hat{b}}=-\mathcal{N}_{\hat{b}\hat{a}}\,,\qquad \mathcal{C}_{\hat{a}}{}^{ij}=\mathcal{C}_{\hat{a}}{}^{ji}\,. \tag{7}$$

Their reality properties are

$$\overline{\mathcal{S}^{ij}}=\mathcal{S}_{ij}\,,\qquad \overline{\mathcal{X}_{\hat{a}\hat{b}}}=\mathcal{X}_{\hat{a}\hat{b}}\,,\qquad \overline{\mathcal{N}_{\hat{a}\hat{b}}}=\mathcal{N}_{\hat{a}\hat{b}}\,,\qquad \overline{\mathcal{C}_{\hat{a}}{}^{ij}}=\mathcal{C}_{\hat{a}ij}\,. \tag{8}$$

The covariant derivatives obey the (anti)commutation relations [8]:

$$\begin{aligned}\{\mathcal{D}^i_{\hat{\alpha}},\mathcal{D}^j_{\hat{\beta}}\} &= -2\mathrm{i}\,\varepsilon^{ij}\mathcal{D}_{\hat{\alpha}\hat{\beta}}-\mathrm{i}\,\varepsilon_{\hat{\alpha}\hat{\beta}}\varepsilon^{ij}\mathcal{X}^{\hat{c}\hat{d}}M_{\hat{c}\hat{d}}+\frac{\mathrm{i}}{4}\varepsilon^{ij}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Gamma_{\hat{a}})_{\hat{\alpha}\hat{\beta}}\mathcal{N}_{\hat{b}\hat{c}}M_{\hat{d}\hat{e}}\\ &\quad-\frac{\mathrm{i}}{2}\varepsilon^{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma_{\hat{a}\hat{b}})_{\hat{\alpha}\hat{\beta}}\mathcal{C}_{\hat{c}}{}^{ij}M_{\hat{d}\hat{e}}+4\mathrm{i}\,\mathcal{S}^{ij}M_{\hat{\alpha}\hat{\beta}}+3\mathrm{i}\,\varepsilon_{\hat{\alpha}\hat{\beta}}\varepsilon^{ij}\mathcal{S}^{kl}J_{kl}\\ &\quad-\mathrm{i}\,\varepsilon^{ij}\mathcal{C}_{\hat{\alpha}\hat{\beta}}{}^{kl}J_{kl}-4\mathrm{i}\Big(\mathcal{X}_{\hat{\alpha}\hat{\beta}}+\mathcal{N}_{\hat{\alpha}\hat{\beta}}\Big)J^{ij}\,, \end{aligned}\tag{9a}$$

$$\begin{aligned}[\mathcal{D}_{\hat{a}},\mathcal{D}^j_{\hat{\beta}}] &= \frac{1}{2}\Big((\Gamma_{\hat{a}})_{\hat{\beta}}{}^{\hat{\gamma}}\mathcal{S}^j{}_k-\mathcal{X}_{\hat{a}\hat{b}}(\Gamma^{\hat{b}})_{\hat{\beta}}{}^{\hat{\gamma}}\delta^j_k-\frac{1}{4}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}\mathcal{N}^{\hat{d}\hat{e}}(\Sigma^{\hat{b}\hat{c}})_{\hat{\beta}}{}^{\hat{\gamma}}\delta^j_k+(\Sigma_{\hat{a}}{}^{\hat{b}})_{\hat{\beta}}{}^{\hat{\gamma}}\mathcal{C}_{\hat{b}}{}^j{}_k\Big)\mathcal{D}^k_{\hat{\gamma}}\\ &\quad+\text{curvature terms}\,. \end{aligned}\tag{9b}$$

The dimension-1 components of the torsion, $\mathcal{S}^{ij}$, $\mathcal{X}_{\hat{a}\hat{b}}$, $\mathcal{N}_{\hat{a}\hat{b}}$ and $\mathcal{C}_{\hat{a}}{}^{ij}$, enjoy some additional differential constraints implied by the Bianchi identities [8].

Let $D_{\hat{A}}=(D_{\hat{a}},D^i_{\hat{\alpha}})$ be another set of covariant derivatives satisfying the constraints (6), with S^{ij}, $X_{\hat{a}\hat{b}}$, $N_{\hat{a}\hat{b}}$ and $C_{\hat{a}}{}^{ij}$ being the corresponding dimension-1 components of the torsion. The supergeometries, which are associated with $\mathcal{D}_{\hat{A}}$ and $D_{\hat{A}}$, describe the same Weyl multiplet if they are related by a super-Weyl transformation[2] [8] of the form:

$$\mathcal{D}^i_{\hat{\alpha}}=\mathrm{e}^{\sigma}\Big(D^i_{\hat{\alpha}}+4(D^{\hat{\beta}i}\sigma)M_{\hat{\alpha}\hat{\beta}}-6(D_{\hat{\alpha}j}\sigma)J^{ij}\Big)\,, \tag{10a}$$

$$\begin{aligned}\mathcal{D}_{\hat{a}} &= \mathrm{e}^{2\sigma}\Big(D_{\hat{a}}+\mathrm{i}(\Gamma_{\hat{a}})^{\hat{\gamma}\hat{\delta}}(D^k_{\hat{\gamma}}\sigma)D_{\hat{\delta}k}-2(D^{\hat{b}}\sigma)M_{\hat{a}\hat{b}}+\frac{\mathrm{i}}{4}(\Gamma_{\hat{a}})^{\hat{\gamma}\hat{\delta}}(D^k_{\hat{\gamma}}D^l_{\hat{\delta}}\sigma)J_{kl}\\ &\quad+\frac{\mathrm{i}}{2}\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}\hat{e}}(\Sigma^{\hat{b}\hat{c}})_{\hat{\gamma}\hat{\delta}}(D^{\hat{\gamma}k}\sigma)(D^{\hat{\delta}}_k\sigma)M^{\hat{d}\hat{e}}+\frac{5\mathrm{i}}{2}(\Gamma_{\hat{a}})^{\hat{\gamma}\hat{\delta}}(D^k_{\hat{\gamma}}\sigma)(D^l_{\hat{\delta}}\sigma)J_{kl}\Big)\,. \end{aligned}\tag{10b}$$

The components of the torsion are related as follows:

$$\mathcal{X}_{\hat{c}\hat{d}}=\mathrm{e}^{2\sigma}\Big(X_{\hat{c}\hat{d}}-\frac{\mathrm{i}}{2}(\Sigma_{\hat{c}\hat{d}})_{\hat{\gamma}\hat{\delta}}(D^{\hat{\gamma}k}D^{\hat{\delta}}_k\sigma)-3\mathrm{i}(\Sigma_{\hat{c}\hat{d}})_{\hat{\gamma}\hat{\delta}}(D^{\hat{\gamma}k}\sigma)(D^{\hat{\delta}}_k\sigma)\Big)\,, \tag{11a}$$

$$\mathcal{N}_{\hat{c}\hat{d}}=\mathrm{e}^{2\sigma}\Big(N_{\hat{c}\hat{d}}-\mathrm{i}(\Sigma_{\hat{c}\hat{d}})_{\hat{\gamma}\hat{\delta}}(D^{\hat{\gamma}k}D^{\hat{\delta}}_k\sigma)-6\mathrm{i}(\Sigma_{\hat{c}\hat{d}})_{\hat{\gamma}\hat{\delta}}(D^{\hat{\gamma}k}\sigma)(D^{\hat{\delta}}_k\sigma)\Big)\,, \tag{11b}$$

$$\mathcal{C}_{\hat{a}}{}^{jk}=\mathrm{e}^{2\sigma}\Big(C_{\hat{a}}{}^{jk}+\mathrm{i}(\Gamma_{\hat{a}})^{\hat{\alpha}\hat{\beta}}(D^{(j}_{\hat{\alpha}}D^{k)}_{\hat{\beta}}\sigma)-2\mathrm{i}(\Gamma_{\hat{a}})^{\hat{\alpha}\hat{\beta}}(D^{(j}_{\hat{\alpha}}\sigma)(D^{k)}_{\hat{\beta}}\sigma)\Big)\,, \tag{11c}$$

$$\mathcal{S}^{ij}=\mathrm{e}^{2\sigma}\Big(S^{ij}+\frac{\mathrm{i}}{2}(D^{\hat{\gamma}(i}D^{j)}_{\hat{\gamma}}\sigma)-3\mathrm{i}(D^{\hat{\gamma}(i}\sigma)(D^{j)}_{\hat{\gamma}}\sigma)\Big)\,. \tag{11d}$$

[2] In [8], only the infinitesimal super-Weyl transformation was explicitly given.

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Consider the super-Weyl tensor [8]

$$\mathcal{W}_{\hat{a}\hat{b}} := \mathcal{X}_{\hat{a}\hat{b}} - \frac{1}{2}\mathcal{N}_{\hat{a}\hat{b}} \ . \tag{12}$$

It follows from eqs. (11a) and (11b) that it transforms homogeneously,

$$\mathcal{W}_{\hat{a}\hat{b}} = \mathrm{e}^{2\sigma}\, W_{\hat{a}\hat{b}} \ . \tag{13}$$

If the supergeometry $D_{\hat{A}}$ is such that its super-Weyl tensor vanishes, $W_{\hat{a}\hat{b}} = 0$, the same property holds for the supergeometry $\mathcal{D}_{\hat{A}}$. If the supergeometry $D_{\hat{A}}$ is flat, the supergeometry $\mathcal{D}_{\hat{A}}$ will be called *conformally flat*.

Suppose that the two supergeometries under consideration are such that[3]

$$\mathcal{C}_{\hat{a}}{}^{ij} = C_{\hat{a}}{}^{ij} = 0 \ . \tag{14}$$

Then, it follows from (11c) that the parameter σ is constrained. The relevant constraint can be expressed in the form:

$$D^{(i}_{\hat{\alpha}} D^{j)}_{\hat{\beta}} W_0 - \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}} D^{\hat{\gamma}(i} D^{j)}_{\hat{\gamma}} W_0 = 0\,, \qquad W_0 := \mathrm{e}^{-2\sigma} \ . \tag{15}$$

This is the equation for the *field strength of an Abelian vector multiplet*. In what follows, we will assume the fulfillment of (14).

More generally, consider an arbitrary non-Abelian vector multiplet. Its field strength $\mathcal{W}$ obeys the constraint

$$\mathcal{D}^{(i}_{\hat{\alpha}} \mathcal{D}^{j)}_{\hat{\beta}} \mathcal{W} - \frac{1}{4}\varepsilon_{\hat{\alpha}\hat{\beta}} \mathcal{D}^{\hat{\gamma}(i} \mathcal{D}^{j)}_{\hat{\gamma}} \mathcal{W} = 0 \tag{16}$$

and possesses the super-Weyl transformation

$$\mathcal{W} = \mathrm{e}^{2\sigma} W \ . \tag{17}$$

Associated with the vector multiplet is the composite superfield [8]

$$\mathcal{G}^{ij} := \mathrm{tr}\left\{ \mathrm{i}\, \mathcal{D}^{\hat{\alpha}(i} \mathcal{W} \mathcal{D}^{j)}_{\hat{\alpha}} \mathcal{W} + \frac{\mathrm{i}}{2} \mathcal{W} \mathcal{D}^{ij} \mathcal{W} - 2\mathcal{S}^{ij} \mathcal{W}^2 \right\}, \qquad \mathcal{D}^{ij} := \mathcal{D}^{\hat{\alpha}(i} \mathcal{D}^{j)}_{\hat{\alpha}}\,, \tag{18}$$

which enjoys the equation

$$\mathcal{D}^{(i}_{\hat{\alpha}} \mathcal{G}^{jk)} = 0 \tag{19}$$

and possesses the super-Weyl transformation

$$\mathcal{G}^{ij} = \mathrm{e}^{6\sigma} G^{ij} \ . \tag{20}$$

The explicit expression for W_0, eq. (15), and the super-Weyl transformation law (17) imply

$$\mathcal{W}_0 = 1 \ . \tag{21}$$

[3]As observed in [8], the super-Weyl gauge freedom can always be used to choose the gauge $\mathcal{C}_{\hat{a}}{}^{ij} = 0$.

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Then, it follows from (18) and (20) that

$$\mathcal{G}_0^{ij} = -2\mathcal{S}^{ij} = \mathrm{e}^{6\sigma} G_0^{ij} \ . \tag{22}$$

The supergeometry corresponding to the 5D $\mathcal{N}=1$ anti-de Sitter superspace is characterized by the following conditions [8] (see also [11]):

$$\mathcal{C}_{\hat{a}}{}^{ij} = 0 \,, \qquad \mathcal{X}_{\hat{a}\hat{b}} = \mathcal{N}_{\hat{a}\hat{b}} = 0 \,, \qquad \mathcal{S}^{ij} \neq 0 \ . \tag{23}$$

Then, it follows from the Bianchi identities [8] that $\mathcal{S}^{ij}$ is covariantly constant,

$$\mathcal{D}^k_{\hat{\alpha}} \mathcal{S}^{ij} = 0 \ . \tag{24}$$

As argued in [12], in the family of five-dimenisonal $\mathcal{N}$-extended anti-de Sitter superspaces

$$\mathrm{AdS}^{5|8\mathcal{N}} = \frac{\mathrm{SU}(2,2|\mathcal{N})}{\mathrm{SO}(4,1)\times \mathrm{U}(\mathcal{N})} \,,$$

it is only the case $\mathcal{N}=1$ which corresponds to (locally) conformally flat supergeometry (although no explicit construction was given in [12]). Below we will derive an explicit realization for the 5D $\mathcal{N}=1$ anti-de Sitter superspace as a locally conformally flat supergeometry.

Let us look for a supersymmetric extension of the AdS_5 metric in Poincaré coordinates[4]

$$\mathrm{d}^2 s = \left(\frac{R}{z}\right)^2 \Big(\eta_{mn} \mathrm{d}x^m \mathrm{d}x^n + \mathrm{d}z^2\Big) \,, \qquad R = \mathrm{const} \,, \qquad m = 0,1,2,3 \tag{25}$$

with η_{mn} the four-dimensional Minkowski metric. The bosonic coordinates x^m and z are related to those used in the main body of this paper as $x^{\hat{m}} = (x^m, z)$. Since the supergeometry $D_{\hat{A}}$ is flat, our first problem is to look for a real superfield $W_0(z, \theta^{\hat{\mu}}_i)$ which solves eq. (15) for the vector multiplet field strength in flat superspace. There are at least three ways to address this problem:

(i) brute-force approach;

(ii) harmonic superspace construction;

(iii) projective superspace construction.

In the first case, one starts with a general superfield $W_0(z, \theta^{\hat{\mu}}_i)$ and then tries to satisfy eq. (15). In the second and third approaches, one starts with a useful ansatz for the harmonic or projective prepotential for a 5D $\mathcal{N}=1$ vector multiplet, and then read off the corresponding field strength following the rules given in [13, 14]. In all cases, it is convenient to express the four-component Grassmann coordinates, $\theta^{\hat{\alpha}}_i$, in terms of two-components spinors (see [13] for more details, including the explicit expressions for the 5D gamma-matrices in terms of the sigma-matrices etc.).

$$\theta^{\hat{\alpha}}_i = (\theta^\alpha_i, -\bar{\theta}_{\dot{\alpha} i}) \,, \qquad \theta^i_{\hat{\alpha}} = \begin{pmatrix} \theta^i_\alpha \\ \bar{\theta}^{\dot{\alpha} i} \end{pmatrix} \,, \qquad \overline{\theta^\alpha_i} = \bar{\theta}^i_{\dot{\alpha}} \tag{26}$$

[4]These coordinates are known to cover one-half of the AdS hyperboloid.

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as well as to express the 5D $\mathcal{N}=1$ spinor covariant derivatives $D^i_{\hat\alpha}$ (without central charge) in terms of 4D $\mathcal{N}=2$ spinor covariant derivatives D^i_α and $\bar D_{\dot\alpha i}$ (with central charge) following [13]

$$D^i_{\hat\alpha} = \begin{pmatrix} D^i_\alpha \\ \bar D^{\dot\alpha i} \end{pmatrix}, \qquad D^{\hat\alpha}_i = (D^\alpha_i\,,\,-\bar D_{\dot\alpha i}) \tag{27}$$

where

$$D^i_\alpha = \frac{\partial}{\partial\theta^\alpha_i} + \mathrm{i}\,(\sigma^b)_{\alpha\dot\beta}\,\bar\theta^{\dot\beta i}\,\partial_b + \theta^i_\alpha\partial_z\,, \qquad \bar D_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} - \mathrm{i}\,\theta^\beta_i(\sigma^b)_{\beta\dot\alpha}\,\partial_b - \bar\theta_{\dot\alpha i}\partial_z\,. \tag{28}$$

The most general expression for the field strength $W_0(z,\theta^\alpha_i,\bar\theta^j_{\dot\alpha})$ can be shown to be:

$$\begin{aligned} W_0 = {} & A + \mathrm{i}\big(\theta_{ij}-\bar\theta_{ij}\big)B^{ij} - \frac{1}{12}(\theta^4+\bar\theta^4)\partial_z^2 A + \mathrm{i}\,\theta^k{}_{(i}\bar\theta_{j)k}\partial_z B^{ij} + \frac{1}{2}\theta_{ij}\bar\theta^{ij}\partial_z^2 A \\ & + \frac{\mathrm{i}}{12}\big(\theta^4\bar\theta_{ij}-\theta_{ij}\bar\theta^4\big)\partial_z^2 B^{ij} + \frac{1}{144}\theta^4\bar\theta^4\partial_z^4 A\,, \end{aligned} \tag{29}$$

where

$$\theta_{ij} := \theta^\alpha_i\theta_{\alpha j}\,, \qquad \bar\theta^{ij} := \bar\theta^i_{\dot\alpha}\bar\theta^{\dot\alpha j}\,, \qquad \overline{\theta_{ij}} = \bar\theta^{ij}\,, \qquad \theta^4 := \theta^{ij}\theta_{ij}\,, \qquad \bar\theta^4 := \overline{\theta^4}\,. \tag{30}$$

Here $A(z)$ and $B^{ij}(z) = B^{ji}(z)$ are real functions of z,

$$\overline{A} = A\,, \qquad \overline{B^{ij}} = B_{ij}\,, \tag{31}$$

but otherwise are completely arbitrary.

With W_0 given as in eq. (29), we have satisfied the first constraint in (23). The next problem is to solve the second constraint in (23), $\mathcal{X}_{\hat a\hat b} = 0$ or, equivalently, $\mathcal{N}_{\hat a\hat b} = 0$. Its solution is as follows:

$$A(z) = \frac{R}{z}\,, \qquad B^{ij}(z) = -\frac{R}{2z^2}\,\boldsymbol{s}^{ij}\,, \qquad \boldsymbol{s}^{ij} := \frac{s^{ij}}{\sqrt{\frac{1}{2}s^{ij}s_{ij}}}\,, \tag{32}$$

with

$$R = \text{const}\,, \qquad s^{ij} = s^{ji} = \text{const}\,, \qquad \overline{s^{ij}} = s_{ij}\,. \tag{33}$$

It is a short calculation to demonstrate that the *covariantly constant* torsion $\mathcal{S}^{ij}$ is

$$\mathcal{S}^{ij} = \frac{1}{R}\,\boldsymbol{s}^{ij} \;+\; O(\theta)\,. \tag{34}$$

This completes our explicit realization of $\mathrm{AdS}^{5|8}$ as (locally) conformally flat superspace.

Let us leave $\mathrm{AdS}^{5|8}$ for a while, and discuss the structure of a manifestly supersymmetric action principle in the case of an arbitrary conformally flat superspace. In accordance with the supergravity formulation developed in [8, 10], the supersymmetric action is generated by a *covariant projective supermultiplet* of weight two, $\mathcal{L}^{++}(u^+)$, which is defined

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to be holomorphic with respect to additional isotwistor variables $u^+_i \in \mathbb{C}^2 \setminus \{0\}$. The fact that the Lagrangian is projective and has weight +2, means the following:

$$u^+_i \mathcal{D}^i_{\hat\alpha} \mathcal{L}^{++}(u^+) = 0\,, \qquad \mathcal{L}^{++}(c\,u^+) = c^2\,\mathcal{L}^{++}(u^+)\,, \qquad c \in \mathbb{C}\setminus\{0\}\,, \tag{35}$$

see [8] for more details, including the reality condition of $\mathcal{L}^{++}$, $\widetilde{\mathcal{L}}^{++} = \mathcal{L}^{++}$, with respect to the so-called smile conjugation. The action is

$$S(\mathcal{L}^{++}) = \frac{1}{6\pi}\oint_C (u^+ \mathrm{d}u^+) \int \mathrm{d}^5x\,\mathrm{d}^8\theta\,\mathcal{E}\,\frac{\mathcal{L}^{++}}{(\mathcal{S}^{++})^2}\,, \qquad \mathcal{E}^{-1} = \mathrm{Ber}\left(\mathcal{E}_{\hat A}{}^{\hat M}\right)\,. \tag{36}$$

Here C is a closed integration contour, $\mathcal{S}^{++}(u^+) := \mathcal{S}^{ij}u^+_i u^+_j$ and $(u^+\mathrm{d}u^+) := u^{+i}\mathrm{d}u^+_i$.

Let us choose a coordinate system in which the covariant derivatives $\mathcal{D}_{\hat A}$ are related to the flat global ones, $D_{\hat A}$, according to eqs. (10a)–(10b). We then have

$$\mathcal{E} = \mathrm{e}^{-2\sigma} = W_0\,, \qquad -2\mathcal{S}^{++} = W_0^{-3}G_0^{++}\,, \tag{37}$$

with

$$G_0^{++} := G_0^{ij}u^+_i u^+_j = \mathrm{i}\,D^{\hat\alpha +}W_0 D^+_{\hat\alpha}W_0 + \frac{\mathrm{i}}{2}W_0 D^{\hat\alpha+}D^+_{\hat\alpha}W_0\,, \qquad D^+_{\hat\alpha}G_0^{++} = 0 \tag{38}$$

and $D^+_{\hat\alpha} := D^i_{\hat\alpha}u^+_i$. We also have

$$\mathcal{L}^{++} = W_0^{-3}L^{++}\,, \qquad D^+_{\hat\alpha}L^{++} = 0\,. \tag{39}$$

Here $L^{++}(u^+)$ is a *rigid projective supermultiplet* of weight +2 living in flat 5D $\mathcal{N} = 1$ superspace $\mathbb{R}^{5|8}$.

More generally, if $\mathcal{Q}^{(n)}(u^+)$ is a *covariant projective supermultiplet* of weight n,

$$u^+_i \mathcal{D}^i_{\hat\alpha}\mathcal{Q}^{(n)}(u^+) = 0\,, \qquad \mathcal{Q}^{(n)}(c\,u^+) = c^n\,\mathcal{Q}^{(n)}(u^+)\,, \qquad c\in\mathbb{C}\setminus\{0\}\,, \tag{40}$$

it is generated by a *rigid projective supermultiplet* of weight n, $Q^{(n)}(u^+)$, living in $\mathbb{R}^{5|8}$.

$$\mathcal{Q}^{(n)} = W_0^{-3n/2}Q^{(n)}\,, \qquad D^+_{\hat\alpha}Q^{(n)} = 0\,. \tag{41}$$

The above action turns into[5]

$$S(\mathcal{L}^{++}) = \frac{2}{3\pi}\oint_C (u^+\mathrm{d}u^+)\int \mathrm{d}^5x\,\mathrm{d}^8\theta\,\frac{L^{++}W_0^4}{(G_0^{++})^2}\,. \tag{42}$$

Using the identity [8]

$$D^{(+4)}W_0^4 = \frac{3}{4}(G_0^{++})^2\,, \qquad (D^+)^4 := -\frac{1}{96}\varepsilon^{\hat\alpha\hat\beta\hat\gamma\hat\delta}D^+_{\hat\alpha}D^+_{\hat\beta}D^+_{\hat\gamma}D^+_{\hat\delta}\,, \tag{43}$$

[5]In general, the transformation (10a)–(10b) relating the "flat" and "curved" covariant derivatives, can be defined only locally, as in the case of $\mathrm{AdS}^{5|8}$. Although the locally supersymmetric action (36) is globally defined, its "flat" form (42) holds in general locally. In this paper, we do not discuss global issues.

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we can next transform $S(\mathcal{L}^{++})$ as follows:

$$\begin{aligned} S(\mathcal{L}^{++}) &= \frac{2}{3\pi}\oint_C \frac{(u^+\mathrm{d}u^+)}{(u^+u^-)^4}\int \mathrm{d}^5x\,(D^-)^4(D^+)^4\left\{\frac{L^{++}W_0^4}{(G_0^{++})^2}\right\}\Big|_{\theta=0} \\ &= \frac{1}{2\pi}\oint_C \frac{(u^+\mathrm{d}u^+)}{(u^+u^-)^4}\int \mathrm{d}^5x\,(D^-)^4 L^{++}\Big|_{\theta=0}\ . \end{aligned} \tag{44}$$

Here

$$(D^-)^4 := -\frac{1}{96}\varepsilon^{\hat\alpha\hat\beta\hat\gamma\hat\delta}D^-_{\hat\alpha}D^-_{\hat\beta}D^-_{\hat\gamma}D^-_{\hat\delta}\,, \qquad D^-_{\hat\alpha} := u^-_i D^i_{\hat\alpha}\,, \tag{45}$$

and the isotwistor u^-_i introduced is constrained to obey the inequality $(u^+u^-)\neq 0$ (which means that u^+_i and u^-_i are linearly independent) but otherwise is completely arbitrary.

It is possible to transform the action further and represent it as an integral over 4D $\mathcal{N}=1$ superspace [14, 11]. First of all, we note that the action is invariant under arbitrary projective transformations of the form

$$(u_i{}^-\,,\,u_i{}^+) \;\to\; (u_i{}^-\,,\,u_i{}^+)\,R\,, \qquad R=\begin{pmatrix} a & 0\\ b & c\end{pmatrix} \in \mathrm{GL}(2,\mathbb{C})\ . \tag{46}$$

This symmetry implies that the action is actually independent of u^-_i, and that the isotwistor u^+_i provides homogeneous coordinates for $\mathbb{C}P^1$. Second, without loss of generality, we can assume that the integration contour C does not intersect the north pole of $\mathbb{C}P^1$. We thus can chose

$$u^{+i} = u^{+\underline{1}}(1,\zeta) \equiv u^{+\underline{1}}\zeta^i\,, \qquad u^-_i = (1,0)\,, \tag{47}$$

as well as

$$L^{++}(u^+) = \mathrm{i}(u^{+\underline{1}})^2\zeta\, L(\zeta)\,, \tag{48}$$

with ζ the complex local coordinate parametrizing $\mathbb{C}P^1$. Now, the constraint $D^+_{\hat\alpha}L^{++}=0$ is equivalent to $\zeta^i D_{i\hat\alpha}L(\zeta)=0$. The latter can be used to rewrite (44) in the form:

$$S(\mathcal{L}^{++}) = \frac{1}{2\pi\mathrm{i}}\oint_C\frac{\mathrm{d}\zeta}{\zeta}\int \mathrm{d}^5x\,\mathrm{d}^4\theta\, L(\zeta)\Big|_{\theta^\alpha_{\underline{2}}=0}\ . \tag{49}$$

In this form, the supersymmetric action is given in terms of $\mathcal{N}=1$ superfields.[6]

If the Lagrangian L^{++} is independent of the vector multiplet associated with W_0, then the action (44) contains no information about the curved supergeometry, and thus (44) describes a rigid superconformal theory of the general type studied in [14]. An example of such theories is the general superconformal nonlinear sigma-model formulated in terms of *covariant arctic weight-one* multiplets $\Upsilon^+(u^+)$ and their smile-conjugates $\widetilde{\Upsilon}^+$ and described by the Lagrangian [14, 11, 15]

$$\mathcal{L}^{++} = \mathrm{i}\,K(\Upsilon^+,\widetilde{\Upsilon}^+)\,, \tag{50}$$

[6]Eq. (49) is the 5D $\mathcal{N}=1$ version of the projective superspace action principle [16, 17].

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with $K(\Phi^I, \bar{\Phi}^{\bar{J}})$ a real analytic function of n complex variables Φ^I, where $I = 1, \dots, n$. For $\mathcal{L}^{++}$ to be a weight-two real projective superfield, it is sufficient to require

$$\Phi^I \frac{\partial}{\partial \Phi^I} K(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi}) \ . \tag{51}$$

Let us give an example of dynamical systems with the Lagrangian L^{++} depending on the vector multiplet W_0. Following [11, 9], consider the system of interacting *covariant arctic weight-zero* multiplets $\mathbf{\Upsilon}(u^+)$ and their smile-conjugates $\widetilde{\mathbf{\Upsilon}}$ described by the Lagrangian

$$\mathcal{L}^{++} = \frac{1}{2} \mathcal{S}^{++} \, \mathbf{K}(\mathbf{\Upsilon}, \widetilde{\mathbf{\Upsilon}}) \,, \tag{52}$$

with $\mathbf{K}(\Phi^I, \bar{\Phi}^{\bar{J}})$ a real function which is not required to obey any homogeneity condition. For this model, the line integral in (49) should be carried out around the origin. Because $\mathbf{\Upsilon}(u^+)$ has vanishing weight, $n = 0$, eq. (41) means that $\mathbf{\Upsilon}(u^+) = \mathbf{\Upsilon}(\zeta)$ is a rigid projective supermultiplet. The corresponding flat-superspace form of the Lagrangian is

$$L^{++} = -\frac{1}{4} G_0^{++} \, \mathbf{K}(\mathbf{\Upsilon}, \widetilde{\mathbf{\Upsilon}}) \ . \tag{53}$$

The action can be seen to be invariant under Kähler transformations of the form

$$\mathbf{K}(\mathbf{\Upsilon}, \widetilde{\mathbf{\Upsilon}}) \ \to \ \mathbf{K}(\mathbf{\Upsilon}, \widetilde{\mathbf{\Upsilon}}) + \mathbf{\Lambda}(\mathbf{\Upsilon}) + \bar{\mathbf{\Lambda}}(\widetilde{\mathbf{\Upsilon}}) \,, \tag{54}$$

with $\mathbf{\Lambda}(\Phi^I)$ a holomorphic function.

To describe the dynamics of Yang-Mills supermultiplets, we should introduce a gauge field $V_0(u^+)$ for the Abelian vector multiplet W_0 associated with our conformally flat superspace. The $V_0(u^+)$ is a *tropical weight-zero* multiplet such that the field strength is given as [14]

$$W_0 = \frac{1}{16\pi \mathrm{i}} \oint \frac{(u^+ \mathrm{d}u)}{(u^+ u^-)^2} \, (D^-)^2 \, V_0(u^+) \,, \qquad (D^-)^2 := D^{-\hat{\alpha}} D^-_{\hat{\alpha}} \ . \tag{55}$$

Since V_0 has vanishing weight, $n = 0$, eq. (41) means that $V_0(u^+) = V_0(\zeta)$ is invariant under the super-Weyl transformations, i.e. $\mathcal{V}_0 = V_0$. The field strength W_0 is invariant under the gauge transformations

$$V_0 \ \to \ V_0 + \lambda + \widetilde{\lambda} \,, \tag{56}$$

with $\lambda(u^+)$ an arbitrary *arctic weight-zero* superfield. Let $\mathcal{W}$ be the gauge-covariant field strength of a Yang-Mills supermultiplet, and $\mathcal{V}(u^+)$ is a gauge field (i.e. a tropical weight-zero multiplet taking its values in the Lie algebra of the gauge group). Then, we can construct the covariant projective weight-two multiplet

$$\mathcal{G}^{++}(u^+) := \mathcal{G}^{ij} u^+_i u^+_j \,, \tag{57}$$

with $\mathcal{G}^{ij}$ given in (18). Dynamics of the Yang-Mills supermultiplet can be described by the Lagrangian

$$\mathcal{L}^{++}_{\mathrm{YM}} = \frac{1}{g^2} \, \mathcal{V}_0 \, \mathcal{G}^{++} + \kappa \, \mathcal{G}^{++}_0 \, \mathrm{tr} \mathcal{V} \,, \tag{58}$$

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with g and κ the coupling constants. The corresponding action can be seen to be invariant under the gauge transformations (56). The second term in (58) is a Fayet-Iliopoulos term.

If the Kähler potential $\mathbf{K}(\Phi^I, \bar{\Phi}^{\bar{J}})$ in (53) corresponds to a Kähler manifold with isometries, on can gauge the sigma-model following [20]. In particular, one can generate "massive" sigma-models if the gauging is carried out using the frozen vector multiplet $V_0(\zeta)$.

As follows from (53), all information about the curved superspace geometry is now encoded in $G_0^{++}(u^+) = G_0^{ij}\, u_i^+ u_j^+$. In the case of the anti-de Sitter superspace $\mathrm{AdS}^{5|8}$, this superfield can be shown to be

$$G_0^{++}(u^+) = -\frac{2R^2}{z_c^3}\Bigg\{ \boldsymbol{s}^{++} - \frac{3\mathrm{i}}{z_c}\Big((\theta^+)^2 - (\bar{\theta}^+)^2\Big) - \frac{3}{z_c(u^+u^-)}\Big((\theta^+)^2 + (\bar{\theta}^+)^2\Big)\boldsymbol{s}^{+-} + \frac{12}{z_c^2(u^+u^-)^2}(\theta^+)^2(\bar{\theta}^+)^2\boldsymbol{s}^{--}\Bigg\}\,. \tag{59}$$

Here $\boldsymbol{s}^{\pm\pm} = \boldsymbol{s}^{ij}u_i^\pm u_j^\pm$,

$$z_c = z - \frac{1}{(u^+u^-)}\Big(\theta^+\theta^- + \bar{\theta}^+\bar{\theta}^-\Big)\,, \tag{60}$$

and $\theta_\alpha^\pm = \theta_\alpha^i u_i^\pm$ and $\bar{\theta}_{\dot\alpha}^\pm = \bar{\theta}_{\dot\alpha}^i u_i^\pm$. The variables z_c, θ_α^+ and $\bar{\theta}_{\dot\alpha}^+$, which appear in the right-hand side of (59), are annihilated by $D_{\hat\alpha}^+$, that is, they are analytic in the sense of the 5D $\mathcal{N}=1$ version [13] of the harmonic superspace approach [18, 19]. One can check that G_0^{++} is independent of u^-,

$$\frac{\partial}{\partial u^-}G_0^{++} = 0\,, \tag{61}$$

in spite of the fact that separate contributions to the right-hand side of (59) do depend on u^-.

Let us now represent $G_0^{++}(u^+)$, eq. (59), as

$$G_0^{++}(u^+) = \mathrm{i}(u^{+\underline{1}})^2\zeta\, G_0(\zeta)\,. \tag{62}$$

Instead of giving the complete expression for $G_0(\zeta)$, it is sufficient to consider $G_0(\zeta)$ in the limit of $\theta_{\underline{2}}^\alpha = \bar{\theta}_{\dot\alpha}^{\underline{2}} = 0$, since only this truncated expression for $G_0(\zeta)$ appears in the action (49). Defining

$$\theta^\alpha := \theta_{\underline{1}}^\alpha\,, \qquad \bar{\theta}_{\dot\alpha} := \theta_{\dot\alpha}^{\underline{1}}\,, \tag{63}$$

a short calculation gives

$$G_0(\zeta)|_{\theta_{\underline{2}}=0} = \frac{2\mathrm{i}R^2}{z^3}\Bigg\{\Big(\zeta\boldsymbol{s}^{\underline{11}} - 2\boldsymbol{s}^{\underline{12}} + \frac{1}{\zeta}\boldsymbol{s}^{\underline{22}}\Big) + \frac{3}{z}\theta^2\Big(\boldsymbol{s}^{\underline{11}} - \frac{1}{\zeta}(\boldsymbol{s}^{\underline{12}} + \mathrm{i})\Big) + \frac{3}{z}\bar{\theta}^2\Big(-\boldsymbol{s}^{\underline{22}} + \zeta(\boldsymbol{s}^{\underline{12}} + \mathrm{i})\Big) + \frac{12}{z^2}\theta^2\bar{\theta}^2(\boldsymbol{s}^{\underline{12}} + \mathrm{i})\Bigg\}\,. \tag{64}$$

For completeness, we also give the expression for W_0 in the limit of $\theta_{\underline{2}}^\alpha = \bar{\theta}_{\dot\alpha}^{\underline{2}} = 0$.

$$W_0|_{\theta_{\underline{2}}=0} = \frac{R}{z} - \frac{\mathrm{i}R}{2z^2}\Big(\theta^2\boldsymbol{s}^{\underline{11}} - \bar{\theta}^2\boldsymbol{s}^{\underline{22}}\Big) - \frac{\mathrm{i}R}{z^3}\,\bar{\theta}^2\theta^2(\boldsymbol{s}^{\underline{12}} + \mathrm{i})\,. \tag{65}$$

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Up to an SU(2) rotation, one can always choose $\boldsymbol{s}^{ij}$ to have the form:

$$\boldsymbol{s}^{\underline{11}} = \boldsymbol{s}^{\underline{22}} = 0 \qquad \Longleftrightarrow \qquad \boldsymbol{s}^{\underline{12}} = \pm\mathrm{i}\ . \tag{66}$$

Now, it follows from (64) and (65)

$$\boldsymbol{s}^{\underline{12}} = -\mathrm{i} \qquad \Longrightarrow \qquad W_0|_{\theta_{\underline{2}}=0} = \frac{R}{z}\,, \qquad G_0(\zeta)|_{\theta_{\underline{2}}=0} = -\frac{4R^2}{z^3}\ . \tag{67}$$

It is seen that the superfields $W_0|_{\theta_{\underline{2}}=0}$ and $G_0(\zeta)|_{\theta_{\underline{2}}=0}$ are invariant under the standard 4D $\mathcal{N}=1$ super-Poincaré transformations.

It is not difficult to see that the second solution, $\boldsymbol{s}^{\underline{12}} = \mathrm{i}$, in eq. (66) simply corresponds to the replacement $(\theta^\alpha_{\underline{1}}, \bar\theta^{\underline{1}}_{\dot\alpha}) \to (\theta^\alpha_{\underline{2}}, \bar\theta^{\underline{2}}_{\dot\alpha})$ in the above consideration. In particular, we have

$$\boldsymbol{s}^{\underline{12}} = \mathrm{i} \qquad \Longrightarrow \qquad G_0(\zeta)|_{\theta_{\underline{1}}=0} = \frac{4R^2}{z^3}\ . \tag{68}$$

With the choice (67), the action (49) generated by (53) becomes

$$S = \frac{1}{R}\oint_C \frac{\mathrm{d}\zeta}{2\pi\mathrm{i}\zeta}\int \mathrm{d}^5x\,\mathrm{d}^4\theta\left(\frac{R}{z}\right)^3 \mathbf{K}(\boldsymbol{\Upsilon}, \widetilde{\boldsymbol{\Upsilon}})\ . \tag{69}$$

Here the dynamical variables are

$$\boldsymbol{\Upsilon}(\zeta) = \sum_{n=0}^{\infty}\boldsymbol{\Upsilon}_n\,\zeta^n = \Phi + \zeta\Sigma + \dots\,, \qquad \widetilde{\boldsymbol{\Upsilon}}(\zeta) = \sum_{n=0}^{\infty}\frac{(-1)^n}{\zeta^n}\bar{\boldsymbol{\Upsilon}}_n = \bar\Phi - \frac{1}{\zeta}\bar\Sigma + \dots\,, \tag{70}$$

where the two leading components of $\boldsymbol{\Upsilon}(\zeta)$ are constrained 4D $\mathcal{N}=1$ superfields,

$$\bar D^{\dot\alpha}\,\Phi = 0\,, \qquad -\frac{1}{4}\bar D^2\,\Sigma = \partial_z\,\Phi\ . \tag{71}$$

The other components of $\Upsilon(\zeta)$ are complex unconstrained superfields, and they appear to be non-dynamical (auxiliary) in the model under consideration.

In the free case,

$$\mathbf{K}(\boldsymbol{\Upsilon}, \widetilde{\boldsymbol{\Upsilon}}) = R\,\widetilde{\boldsymbol{\Upsilon}}\,\boldsymbol{\Upsilon}\,, \tag{72}$$

one can easily do the contour integral in (69) to result with

$$S = \int \mathrm{d}^5x\,\mathrm{d}^4\theta\left(\frac{R}{z}\right)^3\left(\bar\Phi\Phi - \bar\Sigma\Sigma\right) \;+\; \dots \tag{73}$$

where the omitted terms involve the auxiliary superfields. The latter terms vanish on the equations of motion for the auxiliary superfields. The quadratic action obtained can be shown to agree (upon implementing a superfield Legendre transformation that converts Σ into a chiral superfield) with the model previously constructed in [21] (see also [22]) by rewriting supersymmetric component actions in AdS_5 in terms of 4D $\mathcal{N}=1$ superfields.

Since the explicit z-dependence in (69) is not accompanied by any ζ-dependence, the auxiliary superfields can be eliminated in the AdS_5 case in the same way it has been done in the flat global case for a large class of nonlinear sigma-models, see e.g. [23].

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To describe off-shell massive hypermultiplets living in $\mathrm{AdS}^{5|8}$, it is necessary to have at our disposal a gauge field $V_0(\zeta)$ that generates the corresponding field strength W_0. Assuming the SU(2) choice (66), one can check that $V_0(\zeta)$ can be chosen to be

$$V_0(\zeta) = \frac{R}{z_c \zeta}\Big(\boldsymbol{\theta}^2(\zeta) - \bar{\boldsymbol{\theta}}^2(\zeta)\Big) + \frac{\mathrm{i}R}{z_c^2\zeta^2}\boldsymbol{\theta}^2(\zeta)\bar{\boldsymbol{\theta}}^2(\zeta)\boldsymbol{s}^{\underline{12}}\,, \tag{74}$$

where

$$\begin{aligned} \boldsymbol{\theta}^\alpha(\zeta) &= -\zeta\theta^\alpha_{\underline{2}} - \theta^\alpha_{\underline{1}}\,, \qquad\qquad \bar{\boldsymbol{\theta}}_{\dot\alpha}(\zeta) = -\zeta\bar\theta^{\underline{1}}_{\dot\alpha} + \bar\theta^{\underline{2}}_{\dot\alpha}\,,\\ z_c &= z + (\theta_{\underline{12}} - \bar\theta^{\underline{12}}) + \zeta(\theta_{\underline{22}} + \bar\theta^{\underline{11}})\,. \end{aligned} \tag{75}$$

The corresponding field strength (55) can be checked to agree with (32). Projecting to the 4D $\mathcal{N}=1$ superfields gives

$$V_0|_{\theta_{\underline{2}}=0} = \frac{R}{z}\left(\frac{1}{\zeta}\theta^2 - \zeta\bar\theta^2\right) + \frac{\mathrm{i}R}{z^2}\theta^2\bar\theta^2(\boldsymbol{s}^{\underline{12}} + \mathrm{i})\,, \tag{76}$$

and therefore

$$\boldsymbol{s}^{\underline{12}} = -\mathrm{i} \qquad \Longrightarrow \qquad V_0|_{\theta_{\underline{2}}=0} = \frac{R}{z}\left(\frac{1}{\zeta}\theta^2 - \zeta\bar\theta^2\right)\,. \tag{77}$$

The massive hypermultiplet Lagrangian is obtained by replacing (72) with

$$\mathbf{K}(\boldsymbol{\Upsilon}, \widetilde{\boldsymbol{\Upsilon}}, V_0) = R\,\widetilde{\boldsymbol{\Upsilon}}\,\mathrm{e}^{mV_0}\boldsymbol{\Upsilon}\,, \tag{78}$$

with m the hypermultiplet mass. This model is invariant under gauge transformations

$$V_0 \;\to\; V_0 + \lambda + \widetilde{\lambda}\,, \qquad \Upsilon \;\to\; \mathrm{e}^{-m\lambda}\Upsilon\,, \tag{79}$$

with the gauge parameter $\lambda(\zeta)$ an arctic superfield. In conclusion, we note that the pre-potential (74) should be used in the Lagrangian (58) to describe the dynamics of the Yang-Mills supermultiplet in $\mathrm{AdS}^{5|8}$.

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References

[1] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[2] R. Altendorfer, J. Bagger and D. Nemeschansky, *Supersymmetric Randall-Sundrum scenario*, *Phys. Rev.* **D 63** (2001) 125025 [hep-th/0003117].

[3] T. Gherghetta and A. Pomarol, *Bulk fields and supersymmetry in a slice of AdS*, *Nucl. Phys.* **B 586** (2000) 141 [hep-ph/0003129].

JHEP06(2008)097

[4] A. Falkowski, Z. Lalak and S. Pokorski, *Supersymmetrizing branes with bulk in five-dimensional supergravity*, *Phys. Lett.* **B 491** (2000) 172 [hep-th/0004093].

[5] I.L. Buchbinder and S.M. Kuzenko, *Ideas and methods of supersymmetry and supergravity or a walk through superspace*, IOP, Bristol U.K. (1998).

[6] T. Kugo and K. Ohashi, *Off-shell $D = 5$ supergravity coupled to matter-Yang-Mills system*, *Prog. Theor. Phys.* **105** (2001) 323 [hep-ph/0010288];
T. Fujita and K. Ohashi, *Superconformal tensor calculus in five dimensions*, *Prog. Theor. Phys.* **106** (2001) 221 [hep-th/0104130].

[7] E. Bergshoeff et al., *Weyl multiplets of $N = 2$ conformal supergravity in five dimensions*, *JHEP* **06** (2001) 051 [hep-th/0104113].

[8] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Super-Weyl invariance in 5D supergravity*, *JHEP* **04** (2008) 032 [arXiv:0802.3953].

[9] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Five-dimensional superfield supergravity*, *Phys. Lett.* **B 661** (2008) 42 [arXiv:0710.3440].

[10] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *5D supergravity and projective superspace*, *JHEP* **02** (2008) 004 [arXiv:0712.3102].

[11] S.M. Kuzenko and G. Tartaglino-Mazzucchelli, *Five-dimensional $N = 1$ AdS superspace: geometry, off-shell multiplets and dynamics*, *Nucl. Phys.* **B 785** (2007) 34 [arXiv:0704.1185].

[12] I.A. Bandos, E. Ivanov, J. Lukierski and D. Sorokin, *On the superconformal flatness of AdS superspaces*, *JHEP* **06** (2002) 040 [hep-th/0205104].

[13] S.M. Kuzenko and W.D. Linch III, *On five-dimensional superspaces*, *JHEP* **02** (2006) 038 [hep-th/0507176].

[14] S.M. Kuzenko, *On compactified harmonic/projective superspace, 5D superconformal theories and all that*, *Nucl. Phys.* **B 745** (2006) 176 [hep-th/0601177].

[15] S.M. Kuzenko, *On superconformal projective hypermultiplets*, *JHEP* **12** (2007) 010 [arXiv:0710.1479].

[16] A. Karlhede, U. Lindström and M. Roček, *Self-interacting tensor multiplets in $N = 2$ superspace*, *Phys. Lett.* **B 147** (1984) 297.

[17] U. Lindström and M. Roček, *New hyperkähler metrics and new supermultiplets*, *Commun. Math. Phys.* **115** (1988) 21; *$N = 2$ super Yang-Mills theory in projective superspace*, *Commun. Math. Phys.* **128** (1990) 191.

[18] A.S. Galperin, E.A. Ivanov, S.N. Kalitsyn, V. Ogievetsky and E. Sokatchev, *Unconstrained $N = 2$ matter, Yang-Mills and supergravity theories in harmonic superspace*, *Class. and Quant. Grav.* **1** (1984) 469.

[19] A.S. Galperin, E.A. Ivanov, V.I. Ogievetsky and E.S. Sokatchev, *Harmonic superspace*, Cambridge University Press, Cambridge U.K. (2001).

[20] S.M. Kuzenko, *On superpotentials for nonlinear σ-models with eight supercharges*, *Phys. Lett.* **B 638** (2006) 288 [hep-th/0602050].

[21] D. Marti and A. Pomarol, *Supersymmetric theories with compact extra dimensions in $N = 1$ superfields*, *Phys. Rev.* **D 64** (2001) 105025 [hep-th/0106256].

JHEP06(2008)097

[22] G. Cacciapaglia, G. Marandella and J. Terning, *Dimensions of supersymmetric operators from AdS/CFT*, arXiv:0802.2946.

[23] M. Arai, S.M. Kuzenko and U. Lindström, *Hyperkähler σ-models on cotangent bundles of Hermitian symmetric spaces using projective superspace*, *JHEP* **02** (2007) 100 [hep-th/0612174]; *Polar supermultiplets, Hermitian symmetric spaces and hyperkähler metrics*, *JHEP* **12** (2007) 008 [arXiv:0709.2633].

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